



18TH EAST ASIAN ACTUARIAL CONFERENCE

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Taipei International Convention Center in Taipei Taiwan

Longevity and Life Annuities Funding in Algeria

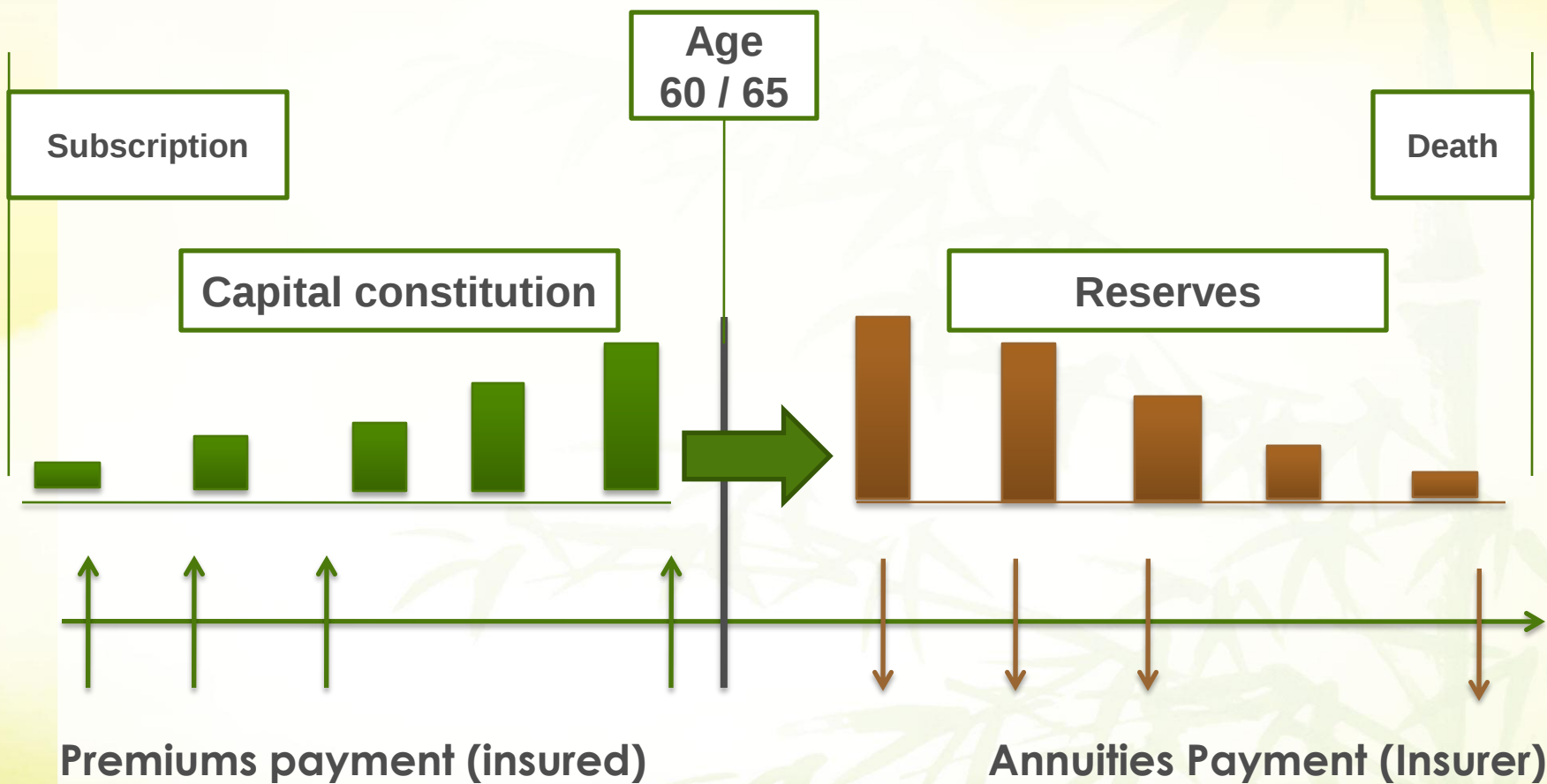
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Introduction



Introduction



Introduction

Static Life-Table :

- One year mortality data
- **Assumption:** Mortality constancy
- In Algeria: Life table 1997 – 99 is still used for life annuities pricing / funding :
- End of the black decade
- High Mortality Level (WP: FLICI & HAMMOUDA, 2012)

Dynamic Life-Table :

- Historical mortality data
- **Assumption:** Mortality decrease

Introduction

- the amount of the Mathematical reserve depends on the life duration
- Now, Mortality is decreasing
- gap between Static life table and mortality evolution
- Dynamic life tables allow consider longevity

Mean Objective

- Construct dynamic life-table for the Algerian population
- But: **how to do for more accuracy?**

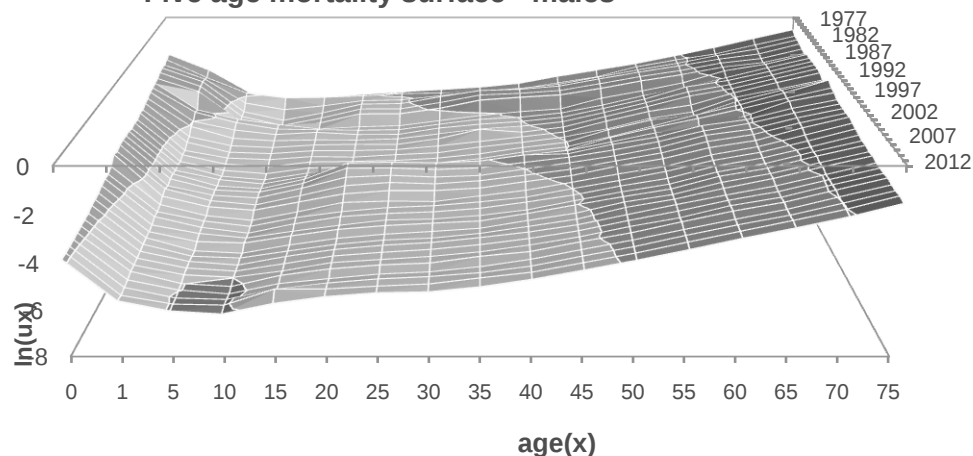
- historical data : Algerian Life-tables as published by the NSO : 1977 - 2013: Males and females (www.ons.dz)

x	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013		
0	0.126	0.11		0.099	0.094	0.083	0.08		0.077		0.062		0.055		0.053		0.053	0.052	0.052	0.052		0.05	0.04	0.035	0.04	0.03	0.03	0.029	0.03	0.03	0.02	0.024	0.023	0.022	0.022	0.021	0.021		
1	0.057	0.045		0.056	0.053	0.048	0.023		0.021		0.014		0.01		0.009		0.009	0.009	0.01	0.009		0.01	0.01	0.006	0.01	0.01	0.005	0.005	0.00	0.00	0.00	0.004	0.004	0.004	0.004	0.004	0.004		
5	0.017	0.019		0.011	0.01	0.01	0.013		0.011		0.009		0.007		0.005		0.005	0.005	0.005	0.005		0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002		
10	0.011	0.011		0.007	0.006	0.006	0.008		0.007		0.007		0.005		0.004		0.004	0.004	0.004	0.004		0.00	0.00	0.002	0.00	0.00	0.002	0.002	0.00	0.00	0.00	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
15	0.013	0.013		0.01	0.01	0.009	0.007		0.007		0.007		0.005		0.005		0.005	0.004	0.005	0.005		0.00	0.00	0.003	0.00	0.00	0.003	0.002	0.00	0.00	0.00	0.002	0.002	0.002	0.002	0.002	0.003	0.002	
20	0.016	0.015		0.014	0.014	0.013	0.01		0.009		0.008		0.006		0.005		0.006	0.006	0.007	0.006		0.00	0.00	0.004	0.00	0.00	0.004	0.003	0.00	0.00	0.00	0.003	0.003	0.002	0.002	0.003	0.002		
25	0.021	0.018		0.016	0.015	0.014	0.01		0.01		0.009		0.008		0.006		0.007	0.007	0.007	0.007		0.01	0.00	0.005	0.00	0.00	0.005	0.004	0.00	0.00	0.00	0.003	0.004	0.003	0.003	0.003	0.003	0.003	
30	0.019	0.022		0.02	0.018	0.016	0.012		0.012		0.01		0.01		0.009		0.008	0.008	0.008	0.009		0.01	0.01	0.006	0.01	0.01	0.006	0.005	0.00	0.00	0.00	0.005	0.005	0.005	0.004	0.004	0.004	0.004	
35	0.018	0.025		0.024	0.023	0.022	0.015		0.015		0.015		0.013		0.011		0.012	0.012	0.012	0.012		0.01	0.01	0.009	0.01	0.01	0.008	0.007	0.01	0.01	0.01	0.007	0.007	0.007	0.006	0.007	0.006		
40	0.021	0.027		0.024	0.023	0.023	0.019		0.017		0.016		0.016		0.015		0.017	0.017	0.018	0.017		0.01	0.01	0.012	0.01	0.01	0.011	0.011	0.01	0.01	0.01	0.01	0.009	0.009	0.008	0.009	0.009	0.009	
45	0.027	0.031		0.027	0.026	0.024	0.023		0.02		0.019		0.019		0.018		0.019	0.019	0.021	0.022		0.02	0.02	0.017	0.02	0.01	0.015	0.014	0.01	0.01	0.01	0.013	0.013	0.013	0.013	0.013	0.013	0.012	
50	0.036	0.041		0.04	0.037	0.035	0.031		0.032		0.03		0.028		0.027		0.03	0.031	0.029	0.029		0.03	0.02	0.022	0.02	0.02	0.021	0.021	0.02	0.02	0.02	0.019	0.02	0.018	0.018	0.018	0.017		
55	0.055	0.062		0.058	0.055	0.052	0.049		0.051		0.051		0.046		0.043		0.047	0.045	0.045	0.044		0.04	0.03	0.032	0.03	0.03	0.028	0.028	0.03	0.03	0.03	0.026	0.03	0.028	0.027	0.027	0.025		
60	0.091	0.076		0.088	0.083	0.079	0.076		0.069		0.076		0.054		0.071		0.05	0.098	0.075	0.078		0.07	0.05	0.054	0.05	0.05	0.049	0.047	0.05	0.04	0.04	0.045	0.045	0.042	0.041	0.042	0.038		
65	0.145	0.102		0.137	0.135	0.129	0.123		0.12		0.109		0.141		0.117		0.099	0.121	0.108	0.107		0.09	0.08	0.079	0.08	0.08	0.074	0.072	0.08	0.07	0.07	0.068	0.068	0.064	0.062	0.064	0.064		
70	0.248	0.16		0.196	0.186	0.177	1		1		1		0.195		0.193		0.193	0.188	0.176	0.173		0.16	0.14	0.135	0.13	0.13	0.125	0.123	0.12	0.11	0.12	0.12	0.123	0.111	0.106	0.109	0.105		
75	0.377	0.314		0.317	0.296	0.277					0.305		0.404				1	1	1	1		0.26	0.23	0.225	0.21	0.21	0.213	0.199	0.21	0.19	0.19	0.193	0.205	0.194	0.19	0.192	0.179		
80	1	1		1	1	1					1		1									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
85																																				1	1	1	1

- Missing calendar years life tables and those were closed out before the age of 80, were completed by another WP (FLICI, 2014 – b)

Data – mortality surfaces

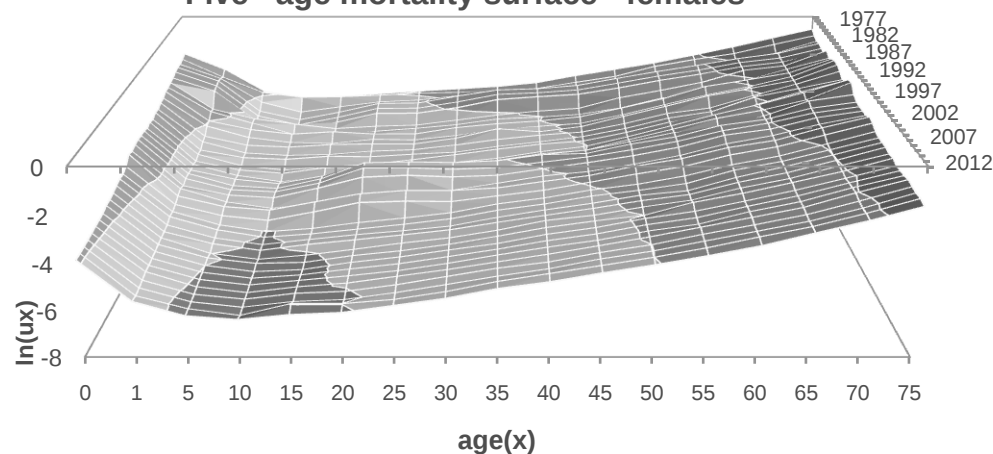
Five age mortality surface - males



Annuitant's portfolio in pay: we consider only the age range (60 years and over)

Single years life table : Karup – king interpolation formula with 03 differences

Five –age mortality surface - females



Ln (Ux)

Methodology

- Single year mortality surface (population aged 60 years and over)
- Lee Carter model LC (Lee & Carter, 1992)
- Lee Carter model with 2 Components LC2 (Renshaw & haberman, 2003 & 2006)
- Lee Carter model with 3 Components LC3 (Booth & al. 2002)
- Lee Carter model with 4 Components LC4
- Mortality trend projecting (ARMA – Siu-Hang Li & al. or Linear: Lee & carter)
- Old ages mortality extending (Coale & Kisker, 1990, Nan Li, 2011)
- Life annuities reserving

Lee Carter Model

- Principal idea of LC Model : extrapolate the historical mortality trend in the future. (Lee and Carter, 1992)
- decomposition on the $\ln$ mortality rate on three components :

$$\ln(\mu_{xt}) = \alpha_x + \beta_x k_t + \varepsilon_{xt}$$

α_x : represents the time average of $\ln$ (at the age x ;

k_t : general mortality trends index ;

β_x : Sensitivity of the age- x mortality compared with the general trend of mortality.

$\varepsilon_{x,t}$: error.

- We work on $\ln(q_{xt})$: Girosi & King, Cairns, 2006, Planchet 2010 $((1-q_x)/q_x)$
- Lead to more accurate results compared with the use of $\ln(U_{xt})$

Multi components Lee Carter Models

■Principal :

- Consider more than one component to reduce the information loss.
- Improve the goodness of the fitting
- Booth & al, 2002 (j=5)
- Renshaw & Haberman, 2003 : two components – Added extra time factor (j=2)
- Cohort effect (renshaw & haberman, 2006)
- Cairns & al. 2006 : old age effect
- Lundestrom & Quist, 2004 (Multi component comparaison – sweden)
- Alho, 2007 : multi factor analysis)

$$\ln(\mu_{xt}) = \alpha_x + \sum_{j=1}^J \beta_x^{(j)} k_t^{(j)} + \varepsilon_{xt}$$

Lee Carter Model - Implimentation

- Estimate alpha-parameter : Average by age of the $\ln(U_x)$

$$\alpha_x = \prod_{t=1}^T \ln(u_{xt})^{\frac{1}{T}}$$

- genarally, we use the global mortality surface to estimate this parameters,
- But, Lee-Carter (1992) introduced the possibility to use only the recents years to escape for some irregularities in historical data.

Lee Carter Model - Implimentation

- Decomposition of the Risidual on two components: Betta and Kappa

$$\ln(u_{x,t}) - \alpha_x = \beta_x * k_t$$

•Two steps decomposition process:

- In the original paper (Lee-Carter, 1992), authors proposed a method based on the SVD (Singular Values Decomposition) by using the Principal Components Analysis (PCA). With as a constraints:

$$\sum_{t=1}^T k_t = 0 \quad \text{and} \quad \sum_{x=1}^X b_x = 1$$

- Wilmouth, 1993, proposed a decomposition process based on Weighted SVD. He considered the survival numbers at each age categorie;
- Theses conditions are not necessary (wilmouth, 1993)

Lee Carter Model - Implimentation

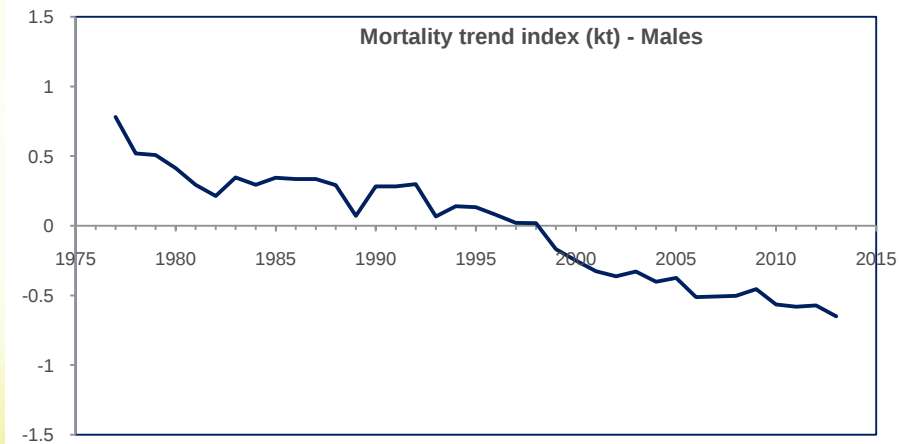
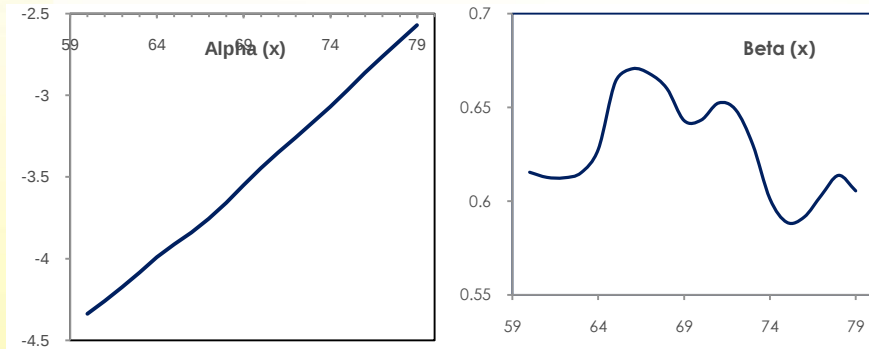
- For the present work, we propose a decomposition process based on Linear programation with XL-Solver
- The optimation criterion is MPAD (Mean Percentage Absolute Deviation on q_x as discussed by (FLICI, 2014: WP):
- The objective function:

$$\text{Min MPAD} = \frac{1}{(T - t) * (X - x)} \sum_{t=1977, x=60}^{2013, 79} |\ln(q_{x,t}) - \exp(\alpha_x - \beta'_x * k'_t)|$$

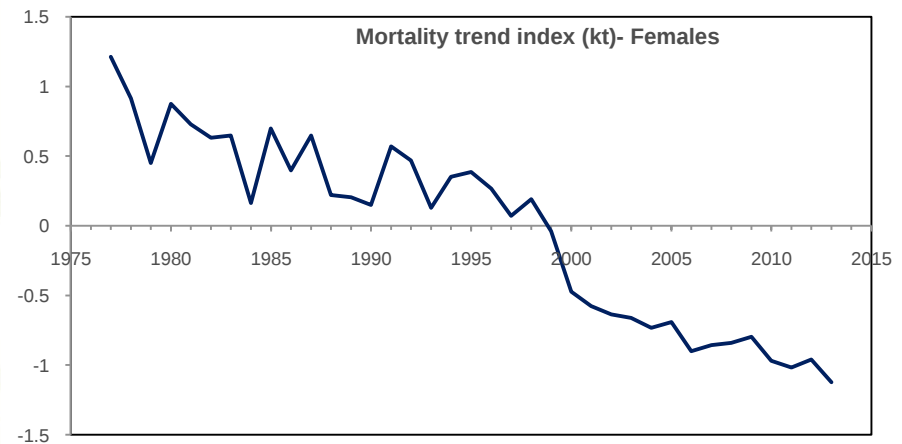
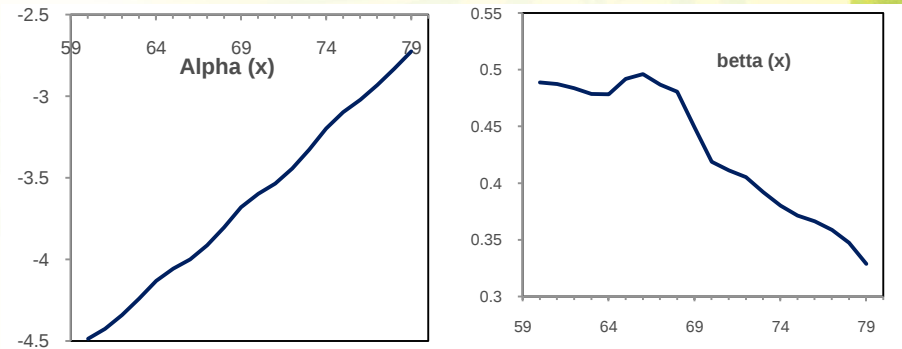
- the ignorance of the constraint allow to improve the fitting quality : the solver need a starting values for k_t and b_x

LC– estimation results

Males



Females



Errors distribution

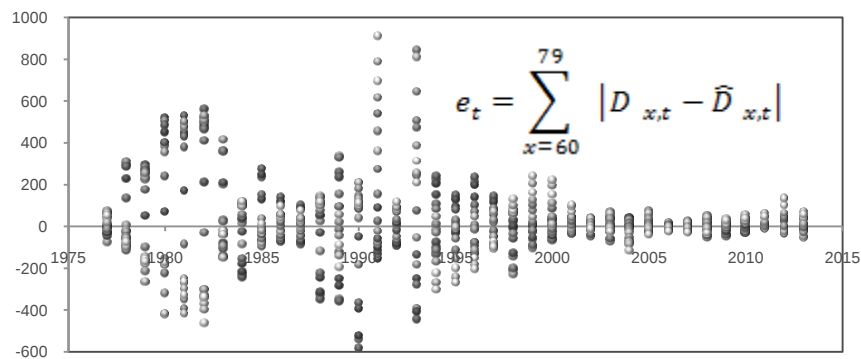
Males

MPAD(x,t)

MPAD	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
60	0.00	14.21	14.27	21.61	24.28	25.89	7.22	12.99	7.49	3.21	2.18	19.00	23.09	35.55	12.38	5.71	21.22	5.71	6.15	6.00	5.49	0.77	0.21	0.75	0.20	1.21	0.01	0.44	2.88	1.49	1.21	1.22	0.00	0.22	0.30	1.51	
61	0.00	14.21	14.27	21.61	24.28	25.89	8.84	10.30	12.24	6.41	4.87	25.10	32.40	49.70	8.40	6.16	27.99	13.40	8.30	12.92	8.40	1.72	0.18	0.49	0.10	1.30	3.00	2.80	2.14	0.70	1.90	3.70	1.43	1.10	3.40	1.50	
62	0.00	12.80	12.21	21.21	21.21	21.27	8.70	16.41	18.42	7.20	1.40	25.70	38.80	55.10	7.27	5.80	40.45	13.10	8.10	13.80	8.70	1.80	0.80	0.70	1.81	2.01	1.80	1.00	2.10	0.10	1.80	3.00	1.30	0.30	2.30	1.50	
63	0.00	16.38	11.61	18.30	19.30	22.00	4.01	13.70	11.70	5.90	4.20	31.30	21.30	36.70	4.42	6.70	34.70	10.91	7.17	11.01	6.90	0.00	0.00	1.77	0.10	1.40	2.80	3.10	1.81	0.20	1.40	3.10	0.90	0.40	2.40	0.60	
64	0.00	4.10	9.80	17.10	18.10	20.70	7.70	8.00	7.00	2.00	1.10	18.70	8.70	26.70	4.00	7.90	20.20	6.60	4.40	4.70	0.00	0.00	1.40	1.00	0.00	1.80	0.01	0.70	2.10	1.00	0.01	0.47	0.00	2.00	0.01	0.10	
65	0.00	14.40	9.70	16.10	18.10	21.47	7.00	5.47	2.70	0.40	1.42	7.43	1.11	12.20	4.11	2.70	12.70	0.00	1.11	0.00	1.01	8.50	6.00	1.80	0.17	1.31	1.80	1.80	0.00	0.07	0.00	0.70	1.20	0.00	0.41	2.80	2.40
66	1.70	0.01	10.80	16.42	21.20	22.90	12.80	2.40	0.00	1.42	2.74	3.80	10.27	2.30	3.47	1.20	9.70	1.60	0.00	4.07	4.00	12.34	5.80	5.71	1.40	2.80	3.01	1.10	0.21	0.30	1.37	0.00	2.82	1.40	0.00	3.80	3.00
67	0.00	3.10	10.70	18.10	18.10	20.10	16.20	0.70	0.41	2.30	3.21	0.01	12.30	1.00	1.02	0.40	5.00	1.00	0.01	5.70	4.00	11.84	6.10	4.10	3.10	3.10	3.40	4.40	0.00	0.00	0.00	1.40	3.40	1.40	0.00	3.10	3.10
68	1.00	2.60	8.07	11.80	16.40	18.10	12.70	0.00	1.10	2.10	3.20	0.01	11.70	0.00	0.00	0.40	6.10	4.90	0.00	0.00	4.00	10.80	5.10	3.10	2.10	2.50	3.40	0.01	0.10	1.41	3.70	3.70	0.70	0.10	2.10	2.10	
69	1.00	1.80	4.80	8.40	16.40	12.20	7.80	0.00	1.10	2.10	2.10	0.01	8.20	0.00	2.10	0.00	4.40	2.47	0.41	2.10	1.80	4.47	3.20	2.40	1.00	1.40	1.40	0.00	1.17	0.00	0.14	1.00	1.70	0.00	0.01	1.27	0.80
70	0.00	0.00	0.01	2.01	4.71	4.71	0.00	0.40	0.40	1.70	1.70	0.00	5.21	0.70	4.07	0.41	14.20	0.00	0.00	0.00	0.00	2.41	1.10	1.40	1.20	0.30	0.80	0.80	0.00	2.10	0.40	0.40	0.00	0.70	0.40	1.40	1.00
71	0.00	0.07	3.20	5.40	5.40	1.01	1.10	1.40	1.40	1.40	1.10	1.70	4.21	2.40	5.10	0.00	19.30	2.47	1.47	2.47	2.47	0.40	0.70	1.10	1.20	1.10	0.30	2.40	0.00	0.00	1.47	1.47	1.27	1.40	1.40	2.10	
72	0.00	0.00	4.80	8.40	7.41	4.80	1.10	1.80	1.80	1.80	0.91	2.40	3.17	3.17	7.80	1.47	21.30	2.40	1.41	2.41	3.10	0.00	0.00	0.80	0.80	1.50	1.50	2.40	0.10	0.10	0.10	0.10	1.77	0.00	1.77	2.40	
73	1.00	0.07	4.80	8.47	7.41	4.80	1.10	1.80	1.80	1.80	0.41	2.47	2.80	3.10	8.40	1.40	30.70	2.40	0.70	2.20	2.40	1.30	0.00	0.00	0.40	0.40	1.70	2.40	0.10	0.10	0.10	0.10	0.10	1.10	1.10	2.10	
74	1.00	0.40	5.40	5.47	1.40	4.80	1.10	1.07	0.80	0.80	0.00	2.40	0.80	3.10	11.81	0.00	17.40	0.01	0.07	0.07	1.00	1.70	1.17	0.00	0.10	1.10	1.80	1.80	0.70	0.01	0.70	0.00	0.80	0.01	0.01	1.40	
75	1.00	0.10	2.00	2.11	3.10	3.10	1.10	1.80	1.80	0.31	0.10	1.67	0.80	1.80	10.10	18.10	18.10	2.40	2.41	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
76	1.00	0.10	1.70	1.80	2.41	3.20	0.40	1.70	1.70	0.01	0.01	0.42	1.10	0.80	0.00	0.70	12.70	1.00	1.00	1.40	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
77	0.00	0.00	2.00	1.11	2.70	4.10	0.00	1.00	1.00	4.10	4.10	2.70	2.01	0.00	0.00	0.00	17.00	2.10	11.70	1.40	1.10	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
78	0.00	1.20	2.70	1.20	3.00	5.10	0.00	0.70	0.21	1.41	0.71	1.21	3.47	0.00	0.00	0.00	20.10	2.00	11.70	7.11	6.21	4.11	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40

MPAD

4.43%

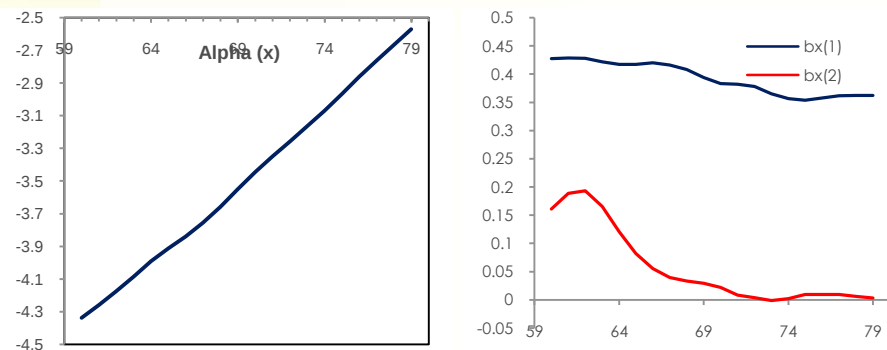


Females

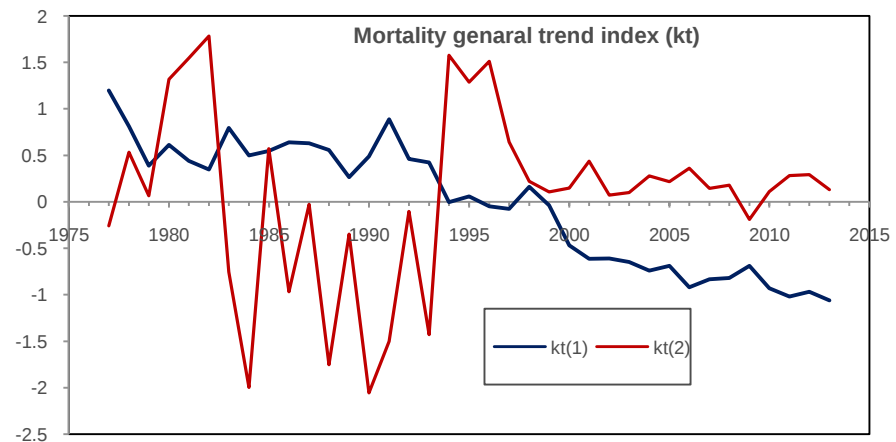
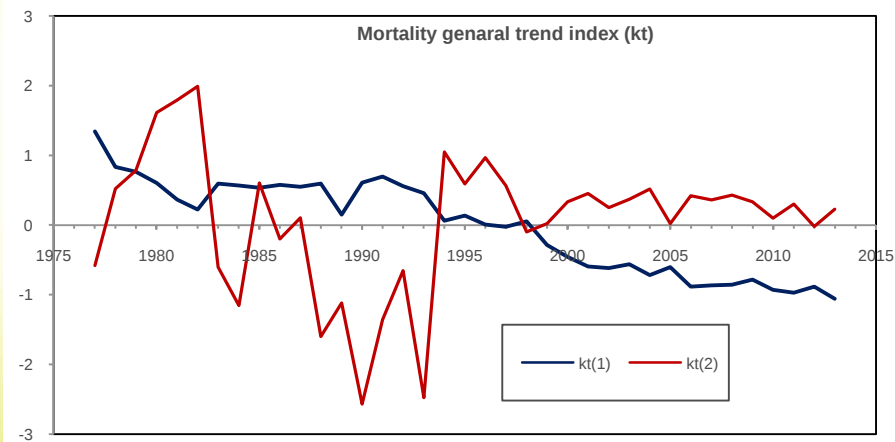
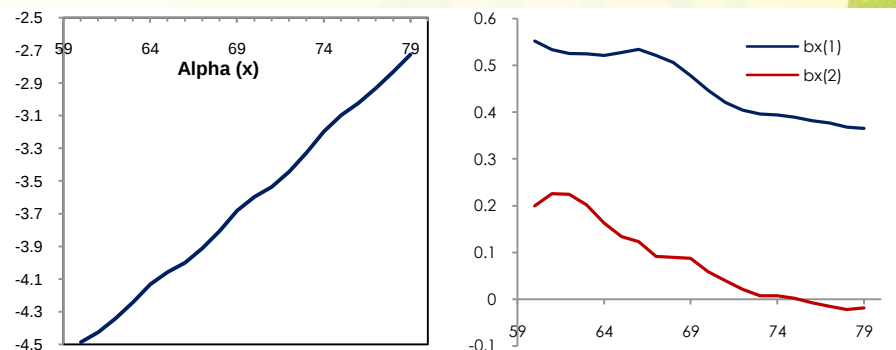
MPAD	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013		
60	0.01	13.01	0.41	16.70	16.20	21.80	5.41	22.27	7.20	0.01	1.30	14.90	12.21	23.21	0.21	0.21	11.47	11.41	7.11	13.17	3.21	3.80	2.10	1.00	1.11	4.47	1.90	0.01	1.80	0.00	1.30	1.20	0.80	2.27	2.80	1.40	1.11		
61	0.00	13.21	0.80	17.31	17.31	19.80	8.21	31.20	11.70	1.27	0.10	23.10	23.10	17.10	0.80	1.00	23.41	16.90	10.10	16.90	4.47	4.80	5.20	3.41	3.41	1.20	2.10	0.00	0.00	0.10	0.10	0.10	1.11	0.10	2.10	2.40	2.41		
62	0.00	11.82	0.40	16.42	16.50	18.12	0.80	16.40	12.40	2.00	0.00	22.17	23.20	16.40	1.80	4.00	20.20	16.40	10.10	17.00	7.01	6.70	5.20	3.40	4.80	1.40	1.90	1.40	0.10	0.10	0.10	0.10	0.10	1.10	0.10	2.70	2.70		
63	0.00	9.10	0.80	16.40	11.00	17.00	2.41	27.00	10.10	4.80	1.20	17.80	12.40	24.20	1.70	2.70	19.40	17.70	8.20	14.11	1.70	4.10	4.80	2.40	1.70	0.90	1.20	2.00	0.07	0.00	0.01	1.10	1.10	0.00	0.70	1.40	1.30		
64	0.01	5.00	1.41	11.80	11.80	14.40	7.40	18.00	6.00	0.00	0.00	10.20	1.11	18.40	2.00	0.00	10.20	1.11	18.40	2.00	0.00	0.00	8.40	1.11	8.40	1.11	0.10	2.10	2.41	0.90	0.00	0.00	0.07	2.10	0.10	0.00	2.10	1.70	
65	1.80	0.07	8.01	10.70	14.20	15.80	10.00	7.00	2.17	1.40	0.10	5.10	8.21	7.50	3.42	2.00	2.01	2.01	1.70	2.21	0.41	1.40	2.10	1.61	1.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70		
66	2.40	2.40	3.80	10.80	15.40	16.50	15.01	3.42	1.01	2.30	0.01	2.40	14.21	3.21	1.41	4.70	1.70	1.90	0.11	1.80	1.40	9.70	4.80	3.80	0.81	1.40	1.72	1.10	1.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40		
67	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40		
68	1.70	4.10	4.10	4.10	4.10	9.40	10.80	10.80	2.81	0.00	0.00	2.10	4.40	1.80	18.40	2.00	0.00	4.10	0.00	0.00	0.00	2.80	1.80	2.80	1.17	8.40	3.70	3.42	0.10	2.41	0.80	0.00	3.41	3.80	1.10	0.11	2.01	0.00	0.10
69	0.70	1.80	0.80	0.17	3.41	4.71	4.10	1.40	1.40	0.20	0.80	1.70	4.10	11.20	5.70	1.40	10.80	0.00	0.00	0.00	0.00	4.80	2.10	1.70	0.21	1.10	0.70	0.80	1.40	1.70	0.80	1.00	0.00	0.40	1.40	1.10	4.10	4.10	
70	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47			
71	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
72	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
73	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
74	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47		
75	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
76	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
77	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
78	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47		
79	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
80	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
81	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
82	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47		
83	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
84	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
85	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
86	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47		
87	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
88	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
89	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
90	1.70	2.80	5.01	1.17	1.70	0.10	5.70	8.20	10.30	0.20	0.20	5.50	0.40	8.21	4.40	4.40	16.70	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47		
91	6.27	3.10	9.27	6.80	6.20	4.11	16.40	10.10	0.00	5.80	10.10	4.40	0.50	11.40	1.70	10.20	20.10	9.20	4.80	6.27	5.01	0.00	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10		
92	5.80	3.10	10.10	9.42	9.50	7.40	15.50	11.11	10.80	3.20	16.70	7.40	7.40	12.00	8.40	7.10	20.40	8.40	5.07	7.50	4.20	3.10	0.80	0.20	0.21	0.40	0.40	1.70	3.10	0.00	1.40	4.50	4.50	0.70	4.20	2.70	0.40		
93	1.40	4.10	4.10	5.80	9.40	13.70	11.40	15.40	0.00	0.01	2.17	1.70	0.11	11.40	0.01	0.10	11.40	0.01	0.10	0.10	10.21	4.10	3.70	0.10	1.70	1.80	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	
94	1.70	2.80	5.01	1.17	1.70	0.1																																	

LC2 – estimation results

Males



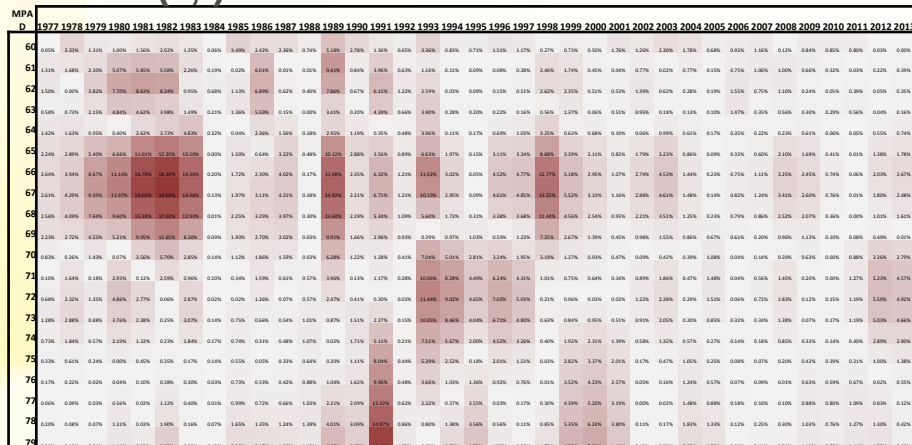
Females



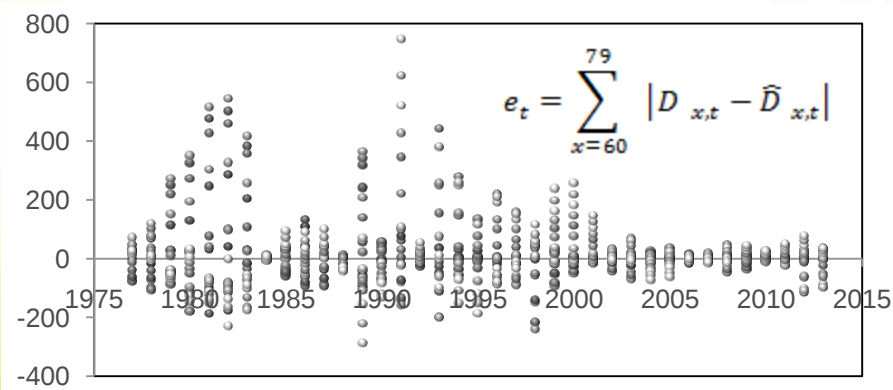
Errors distribution

Males

MPAD(x,t)

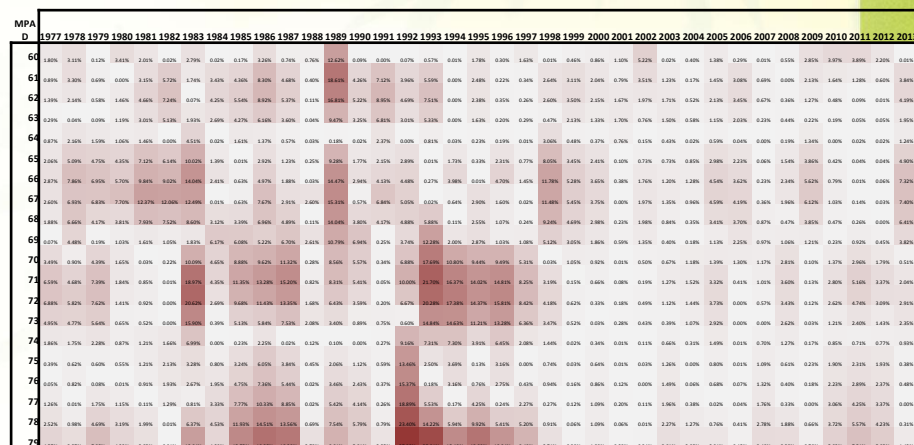


MPAD 2.18%

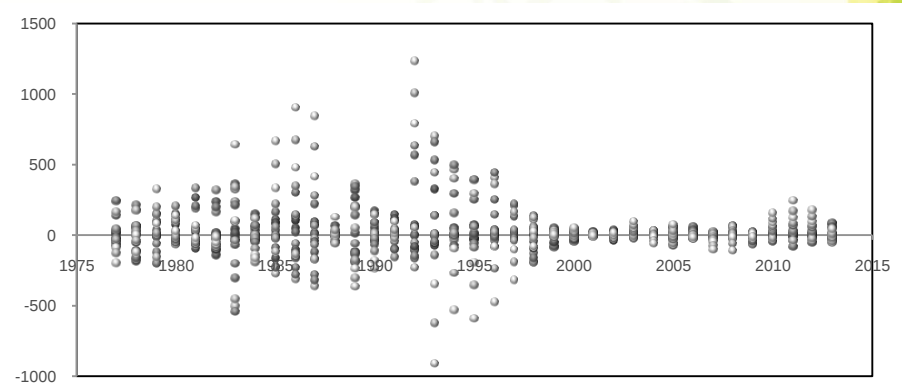


$$e_t = \sum_{x=60}^{79} |D_{x,t} - \bar{D}_{x,t}|$$

Females

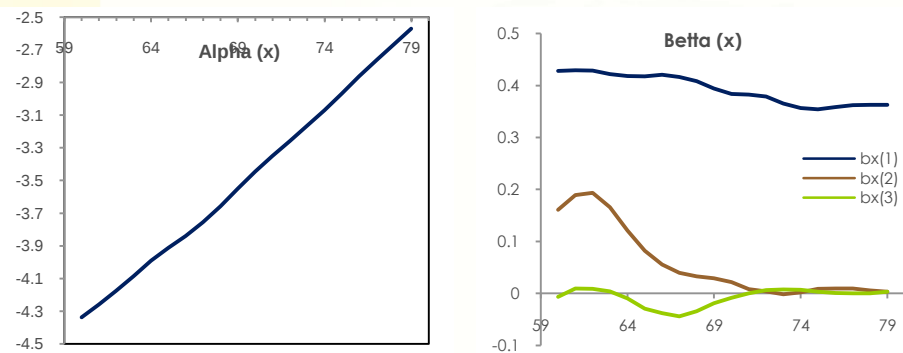


MPAD 3.32%

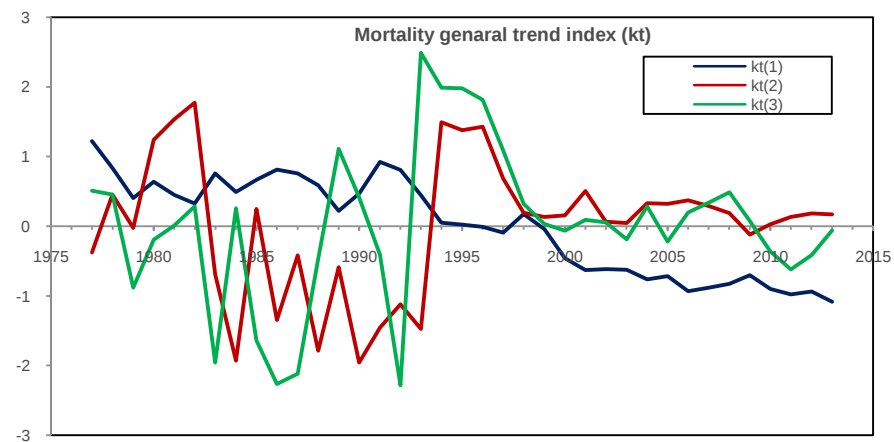
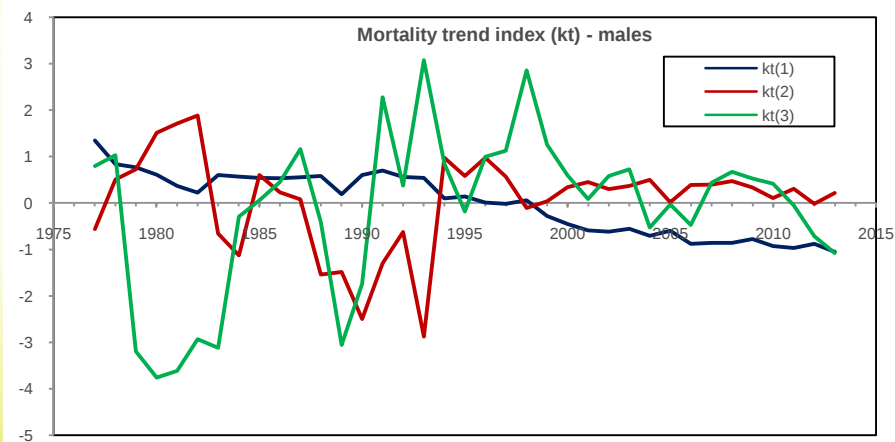
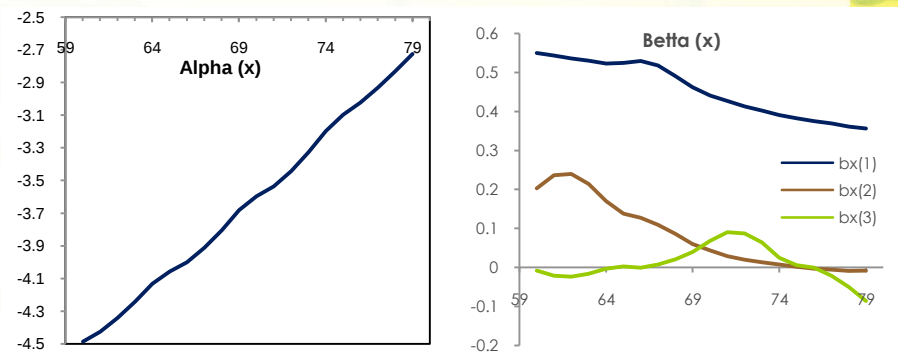


LC3 – estimation results

Males



Females



Females

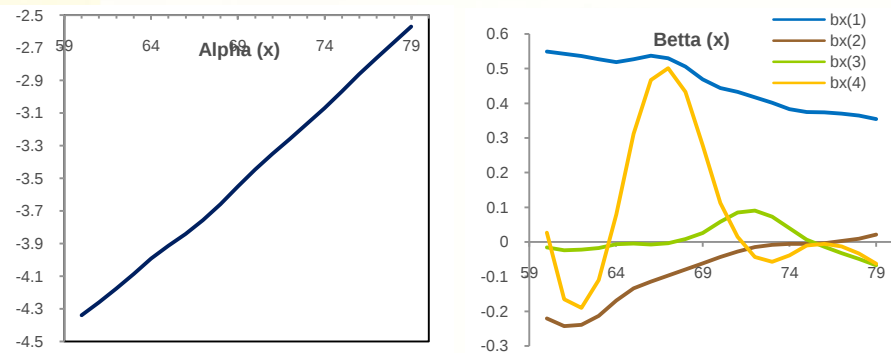
MPAD	1977	1978	1979	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	
60	1.366	0.028	1.118	1.308	1.025	1.195	0.018	1.558	0.005	0.074	3.778	0.025	2.445	0.038	2.438	0.005	0.476	0.005	0.828	0.005	0.005	0.005	2.018	4.868	0.648	0.018	0.388	0.025	0.268	0.038	2.018	3.328	1.688	1.798	0.005	
61	1.488	1.888	1.888	1.888	1.578	1.678	5.838	2.288	1.008	1.788	2.028	0.008	5.628	2.958	4.128	1.888	0.008	1.688	0.008	2.278	2.178	2.178	0.618	2.388	1.218	0.008	2.188	1.758	0.018	1.688	2.028	1.688	1.788	0.018	2.828	
62	1.318	2.128	2.028	4.118	7.808	0.198	3.888	2.428	1.818	3.488	2.368	0.198	4.888	3.388	4.888	0.028	0.158	0.008	2.068	1.888	2.728	2.838	1.488	0.948	0.818	1.758	1.128	3.378	2.128	0.018	2.118	1.588	0.008	0.128	0.228	
63	0.008	0.008	0.708	1.268	1.568	1.898	0.018	1.048	1.288	2.468	1.678	0.008	1.038	1.128	4.728	2.448	0.008	0.318	0.048	0.748	1.338	0.678	0.888	0.028	1.248	0.008	2.268	1.278	0.008	1.008	0.028	0.008	0.008	0.008	1.618	
64	1.438	1.778	0.018	0.028	0.018	1.283	0.098	0.008	1.138	0.188	0.008	1.693	0.738	0.038	1.678	2.348	1.768	0.008	1.168	0.008	3.453	0.888	0.078	0.078	0.318	0.008	0.048	0.048	0.128	0.058	1.778	0.178	0.028	0.268	1.678	
65	1.638	1.328	1.278	1.828	4.008	0.478	16.828	1.018	1.488	1.018	1.918	0.018	16.828	3.128	3.898	0.738	0.038	0.118	0.008	4.618	0.008	3.788	3.128	0.738	0.018	1.688	1.018	2.478	2.228	0.018	1.888	1.608	1.418	0.008	1.118	
66	1.768	7.228	1.118	8.118	9.118	10.128	2.838	8.888	8.128	0.018	10.128	9.118	8.128	9.278	6.318	1.118	0.218	0.008	3.278	1.278	12.118	15.128	5.538	4.838	1.678	1.818	2.238	1.138	8.318	3.118	6.678	2.678	0.008	1.118	7.318	
67	0.018	7.688	1.448	1.448	16.128	16.888	1.008	2.488	0.918	2.488	0.018	16.828	3.128	4.838	0.008	0.718	0.008	7.028	1.678	16.128	16.128	4.838	4.838	1.128	1.868	2.318	1.418	4.388	3.628	4.278	4.838	2.128	1.118	1.218	7.478	
68	1.018	0.418	0.018	0.418	8.478	0.028	18.128	1.888	0.008	0.008	1.888	0.008	18.128	4.838	8.128	0.008	0.008	1.778	1.128	16.128	4.828	4.828	1.268	1.268	2.208	1.228	0.008	2.228	1.228	0.018	2.518	5.228	1.008	1.688	1.468	
69	0.018	0.018	4.0278	3.488	5.868	0.128	18.828	0.418	0.008	1.828	0.008	1.888	0.078	3.328	4.128	0.808	0.228	3.528	0.008	0.008	0.008	0.008	0.008	2.268	0.018	0.078	1.248	0.008	0.008	2.028	1.528	0.008	1.528	1.668	1.008	0.018
70	0.078	1.158	1.158	1.158	2.068	2.068	8.218	0.008	0.008	0.008	0.008	1.888	0																							

MPAD	1.69%
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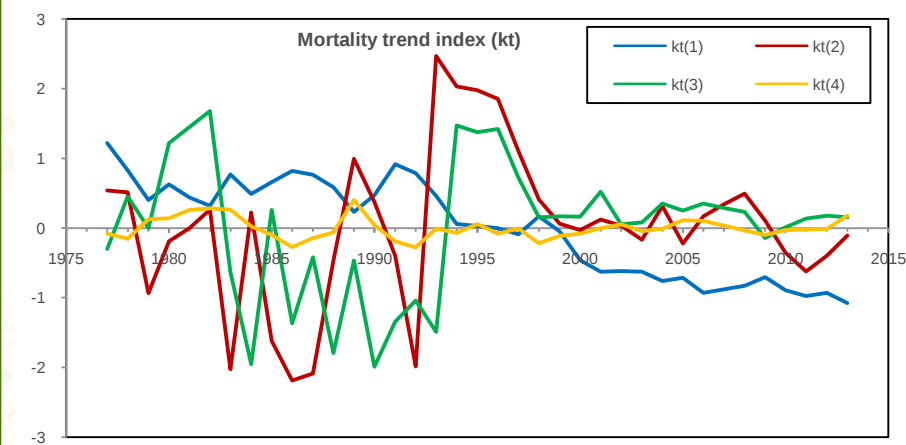
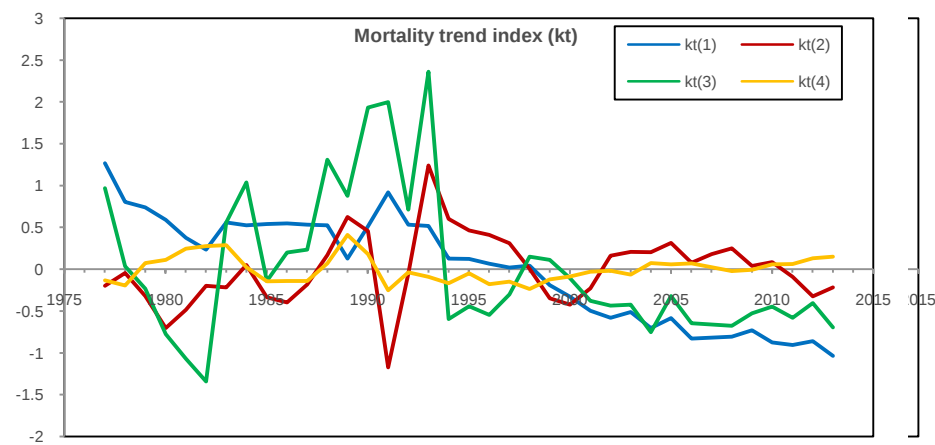
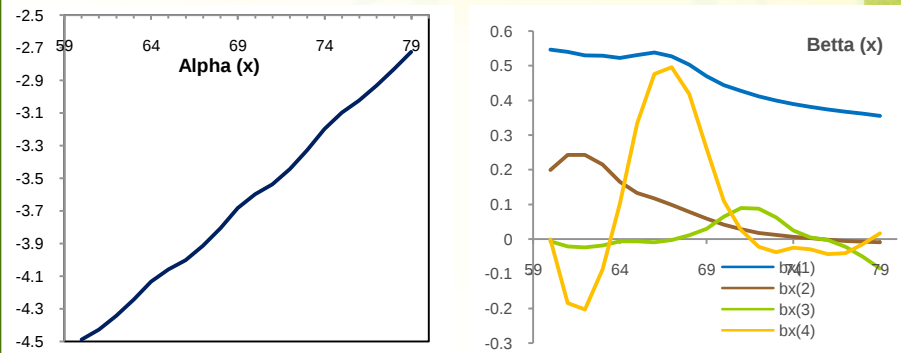
[illegible]

LC4 – estimation results

Males



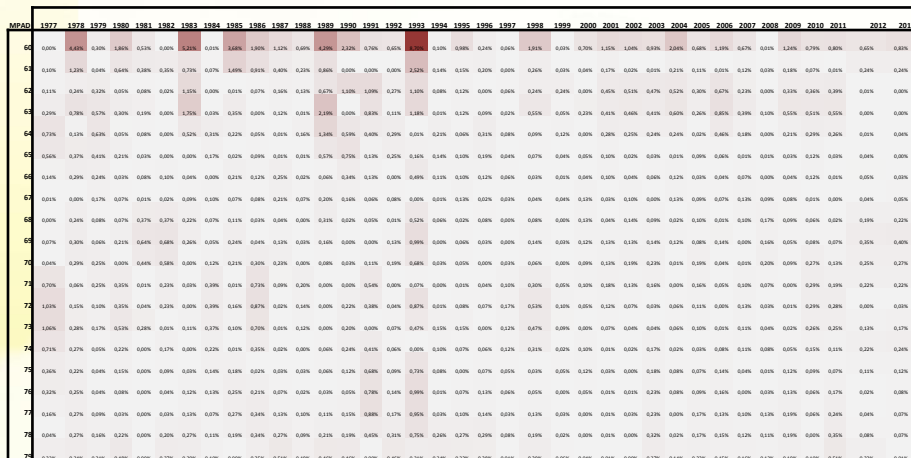
Females



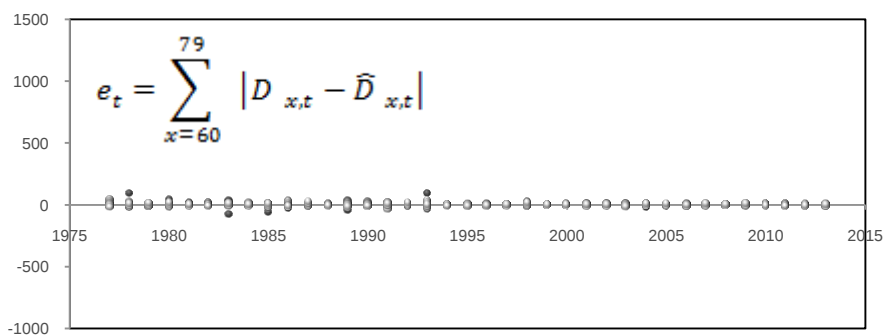
Errors distribution

Males

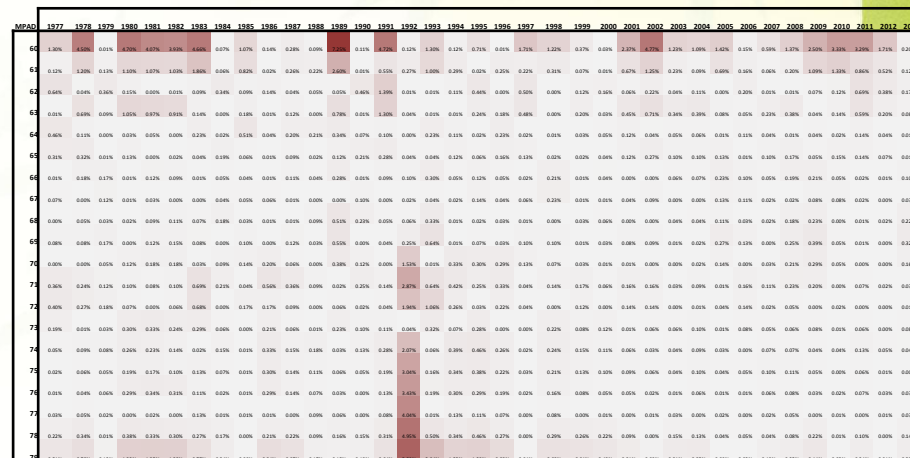
MPAD(x,t)



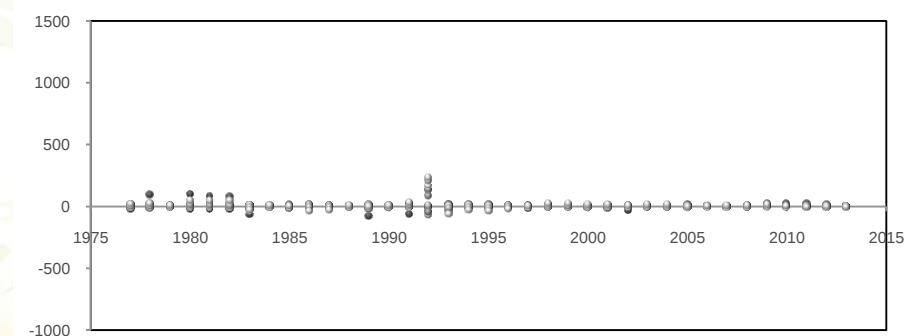
MPAD 0,25%



Females



MPAD 0,29%

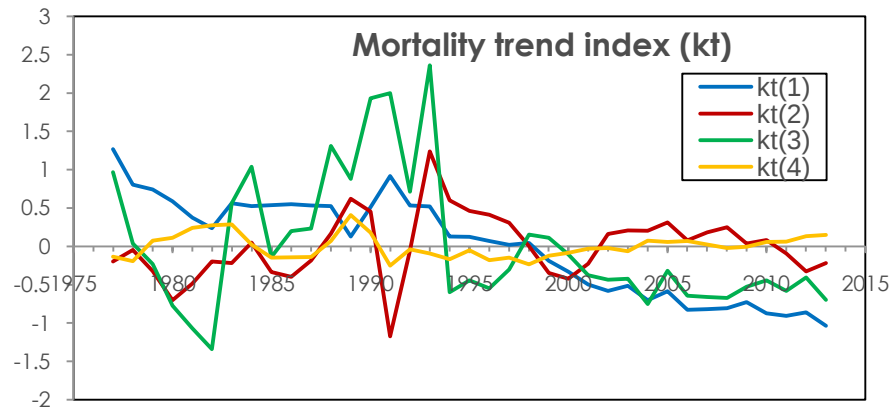


Mortality trend index projection

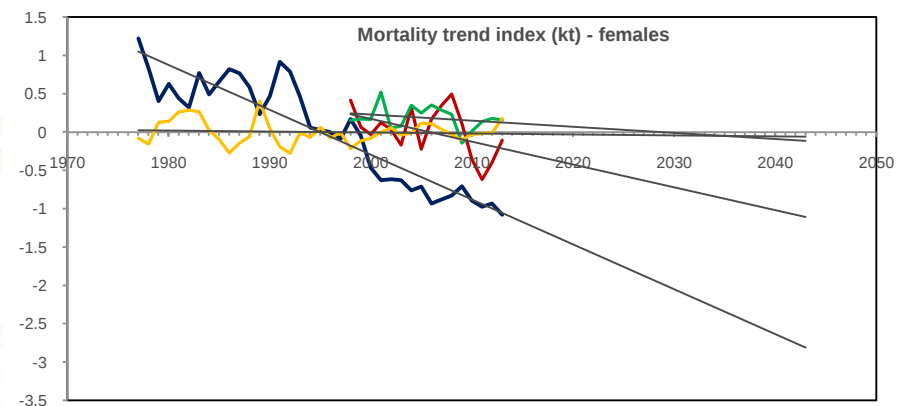
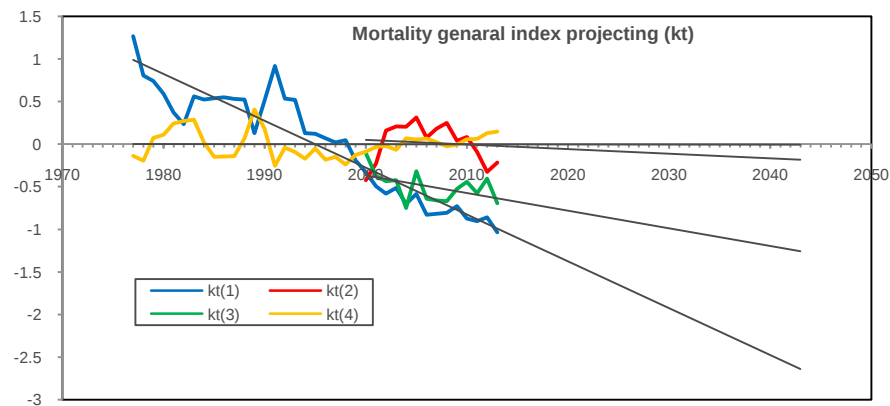
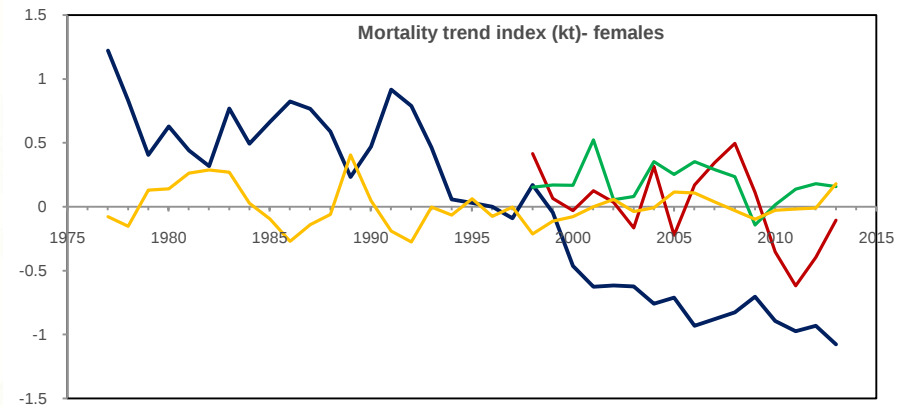
- Lee-Carter (1992) noted that the model would not fit the age-specific mortality data exactly in the jump-off year, they noted that it would be possible to set x equal to the most recently observed log age specific-rates,
- Bell (1997) assessed the performance of several mortality forecasts: LC (as published); LC (with the jump-off year corrected);
- For the Case of the algerian mortality Data, the Black decade (1990-1999) has marked a heigh mortality level wich may greatly affect the general mortality trend
- The extra time components which marque some irregularities during the Black decade, it's only the range [1998 – 2013] that will be considered for the projection

Mortality trend index - projecting

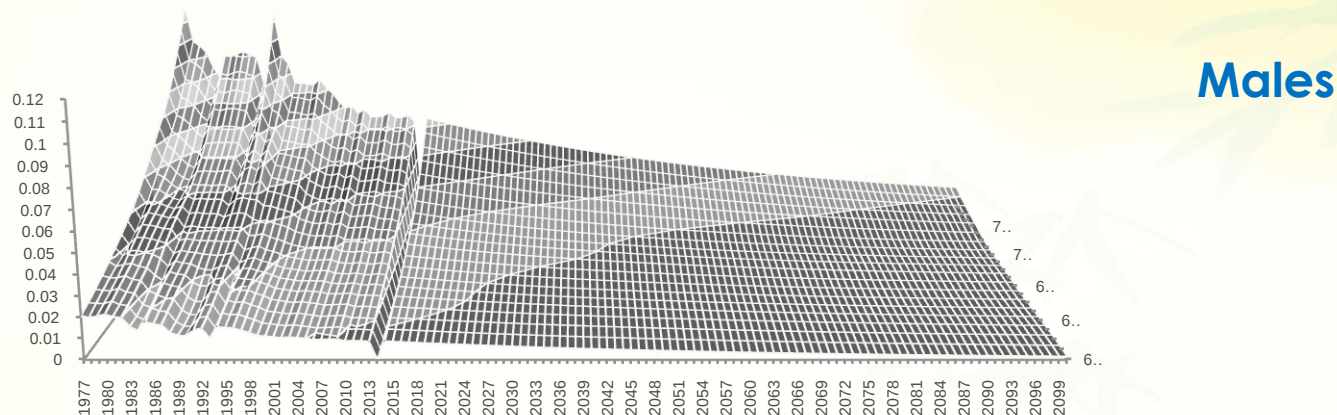
Males



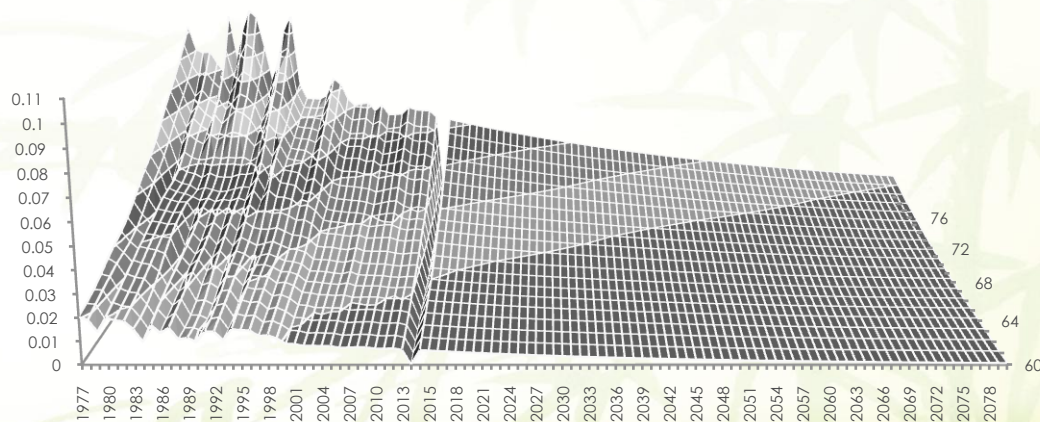
Females



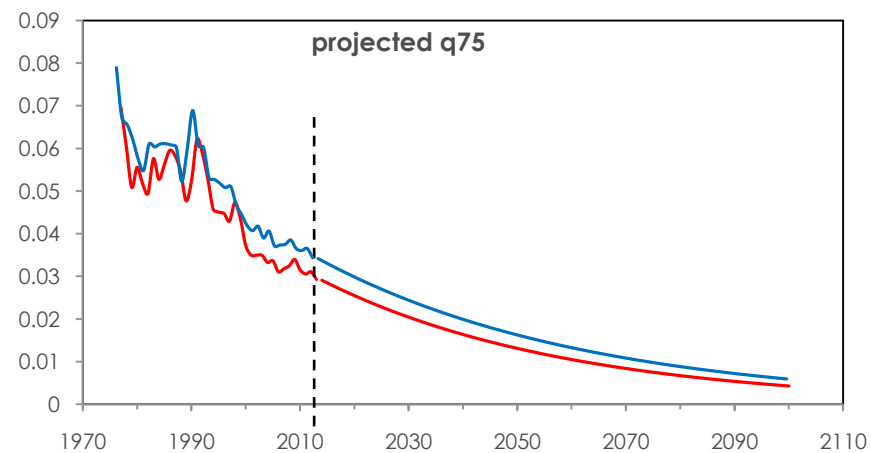
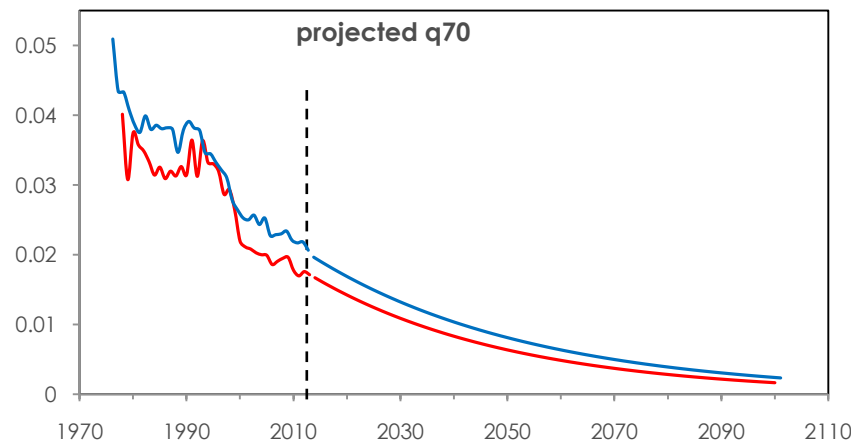
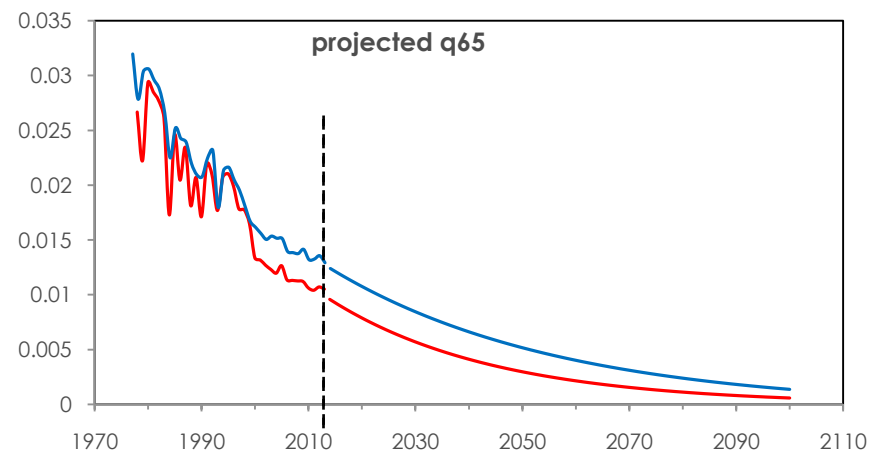
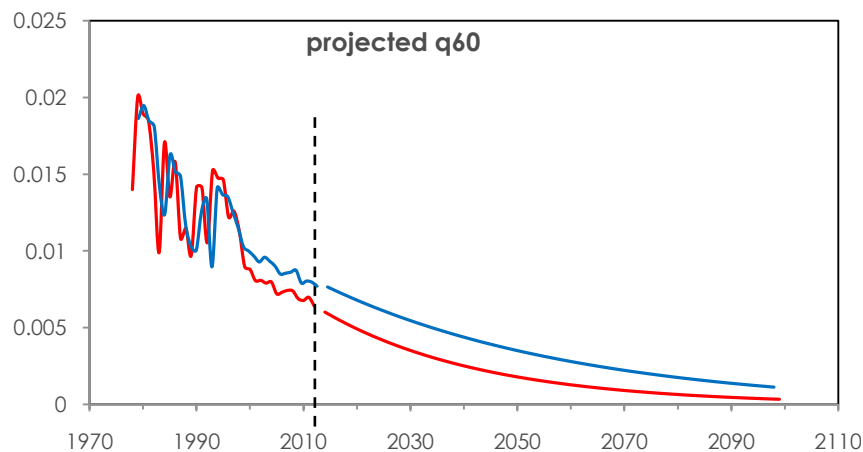
Projected mortality surface



Females



Projected mortality rates



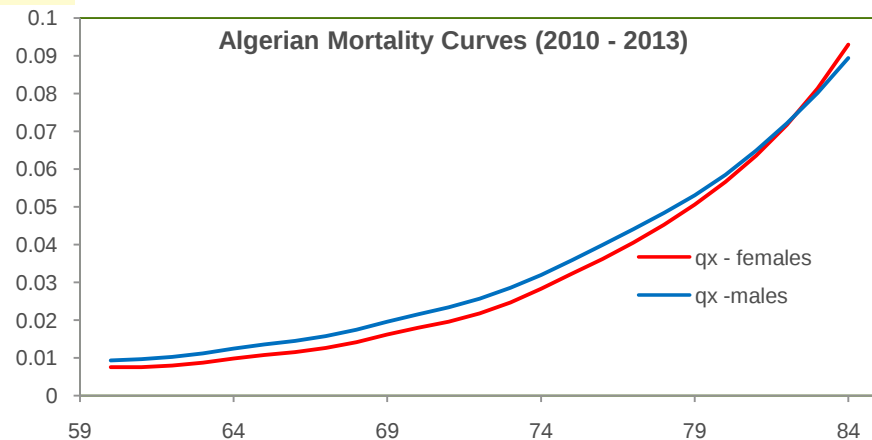
Old Ages Mortality

- Mortality trend changes beyond the age of 80 or 85 (Coale & Guo, 1989) : Specific models were proposed for old ages mortality.
- For developing countries, mortality data at the older ages are unavailable or unreliable
- Several Modal life tables were proposed in this issue, its are used to complete the missing data:
 - for the old age mortality, several model are proposed : **Coale & Guo (1989); Coale & Kisker (1990), Kannisto (1994) Denuit & Gourniaux (2005)**
- All the models which were proposed in this way, have been verified and adjusted on the available data at the older ages
- This data concerns some of the developped countries
- these models, as initially proposed, don't guarantee the quality of the estimated results for the case of the developping countries : Algeria
- Propose a specific (modified) model for the old ages mortality for the algerian population

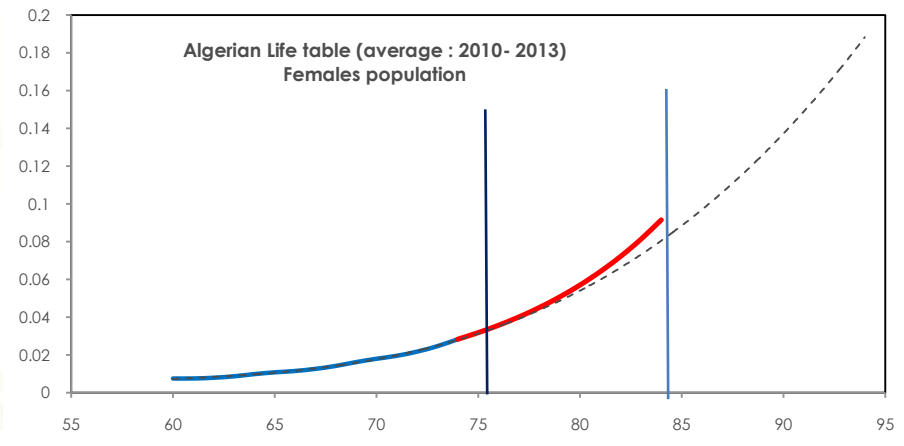
Old ages mortality Models

- It's only starting from 2010, that the algerian life tables were extended until the age groupe [80 -85[years;
- No mortality data beyond the age of 85
- To use this data to predict the mortality trend at the old ages

Mortality curves



Old age extending method



Use the age range [60 – 75 [to adjust the parameters of old ages mortality model

Use the age range [75 – 84[to evaluate and orient the parametrisation of the old age mortality model.

Old ages mortality Models

Extrapolation Formula (Coale & Kisker, 1990)

$$\hat{u}_x = \hat{u}_{x-1} \exp(k_{80} + s \cdot (x - 80)) \quad x = 80, 81, \dots, 109.$$

$$k_{80} = \frac{\ln\left(\frac{\hat{u}_{80}}{\hat{u}_{65}}\right)}{15}$$

$$s = - \frac{\ln\left(\frac{\hat{u}_{79}}{\hat{u}_{110}}\right) + 31k_{80}}{465}$$

$$\hat{u}_{110} = \begin{cases} 1.0 & \text{men} \\ 0.8 & \text{women} \end{cases}$$

The Coale & Kisker model fixes u_{110} to be equal to 0.8 (females) and 1 (males) (Coale & kisker, 1990. Quashie & Denuit, 2005)

In our application, this parameters is defined by the optimisation process.
The same for the K parameter

Old ages mortality Models

Modified formula – Coale & Kisker

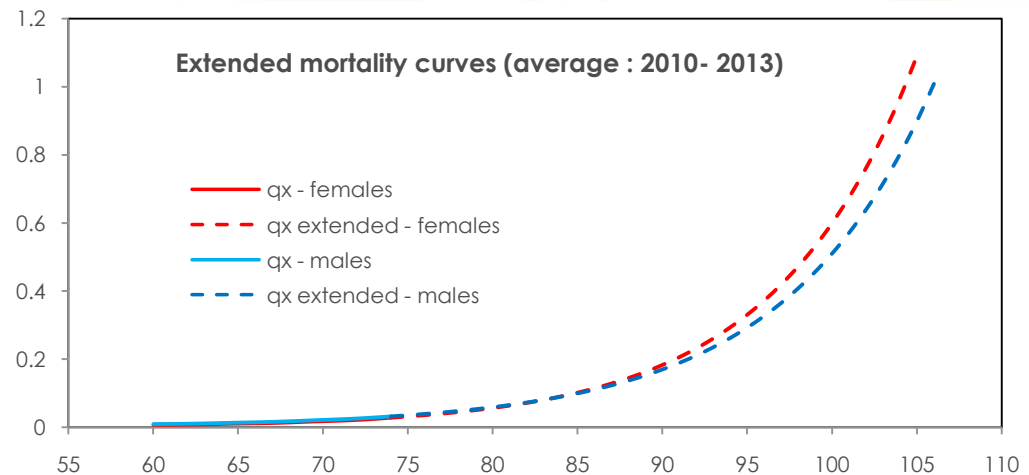
-For our application, we consider:

$$\hat{q}_x = \hat{q}_{x-1} \exp(k_{74} + s \cdot (x - 74)) \quad x = 74, 75, 76, \dots, 109.$$

$$k_{74} = \frac{\ln\left(\frac{\hat{q}_{74}}{\hat{q}_{67}}\right)}{7} \quad s = - \frac{\ln\left(\frac{\hat{u}_{74}}{\hat{u}_{110}}\right) + 36k_{74}}{666}$$

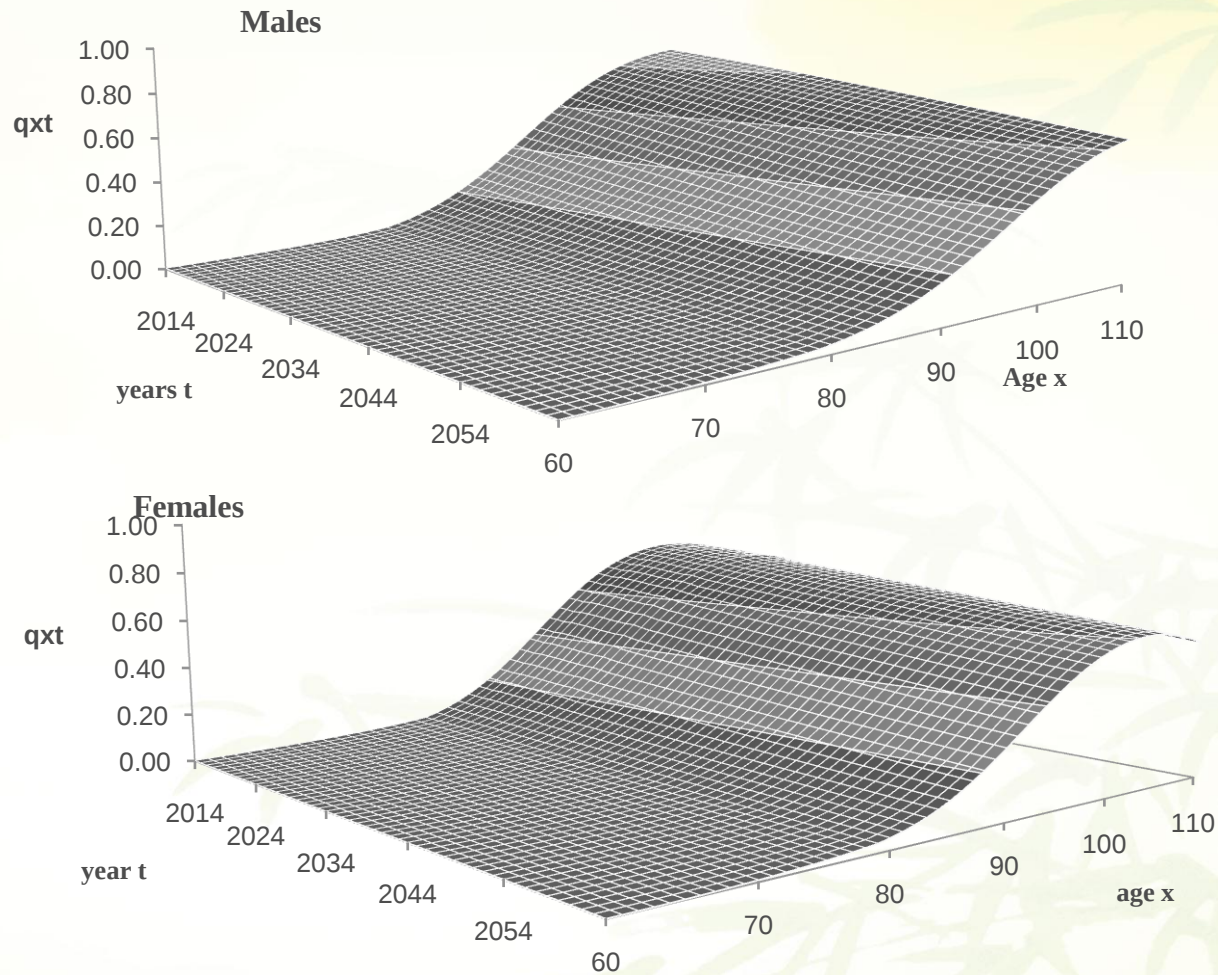
Results

	Males	Females
q110	1,60	1,98
k75	0,1006	0,1153
K- age Range	67- 74	67 - 74
S	0,00043662	0,0001492
MPAD	0,94%	0,90%
Age limit	106	105



Under female mortality beyond the age of 84

Projected mortality surface



Mathematical Reserves calculation

Probability to be alive at time t , for individual x age

$$\hat{P}_{xt} = \exp(-\hat{\mu}_{xt})$$

Probability of survival between the calculation date ' t ' and the annuity payment date $(t+n)$:

$${}_n\hat{P}_{xt} = \prod_{t=0}^{n-1} \hat{P}_{x+i,t+i}$$

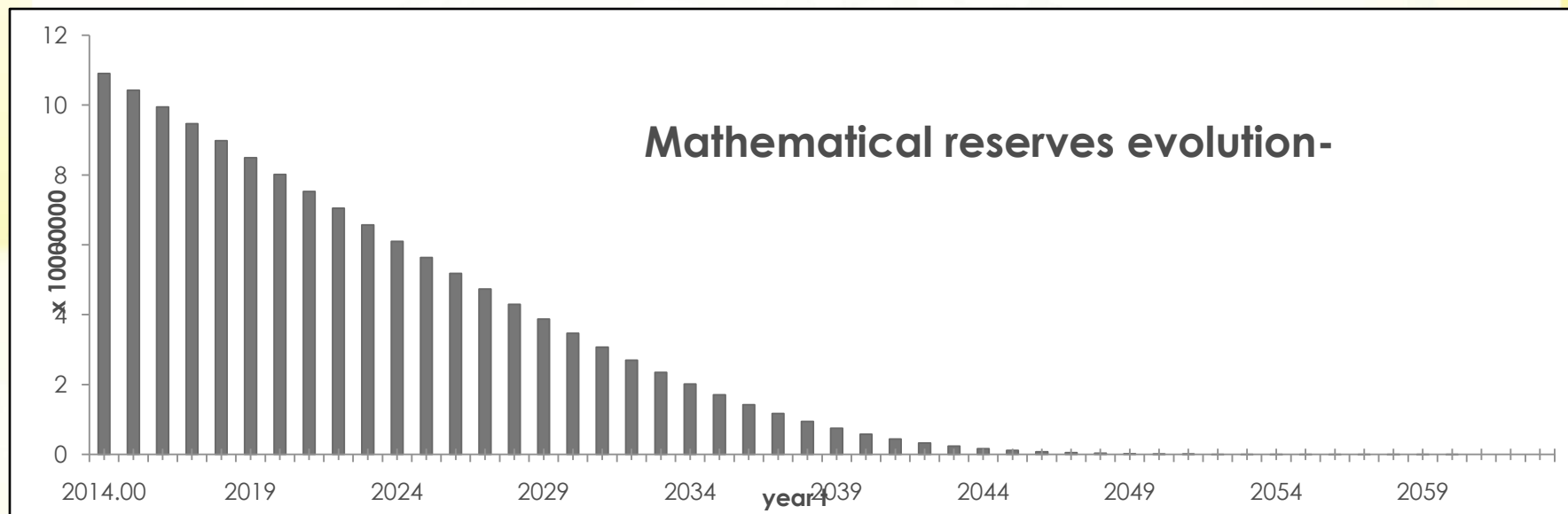
Mathematical reserves for annuitants aged s years old:

$$RM_{st} = N_{st} \cdot R_s \cdot \ddot{a}_x(t)$$

$\ddot{a}_x(t)$ Expected actual value of annuities to pay until the ultimate age

$$\ddot{a}_x(t) = 1 + {}_1\hat{P}_{xt} \cdot V + {}_2\hat{P}_{xt} V^2 + {}_3\hat{P}_{xt} V^3 + \dots + {}_{(w-x-1)}\hat{P}_{xt} V^{w-x-1}$$

Mathematical Reserves



Portfolio : 1400 anuitants in pay

$i = 3\%$ (public bonds – Algeria)

$$RM(2014) = 110\,000\,000 \text{ dzd}$$

Conclusion

- mortality is increasing : it's necessary to use dynamic life table;
- considering the specificities of the algerian data
- Adding extra time components has to improve the fitting quality
- Extending mortality to old ages : information lack beyond the age of 85.
- Under female mortality at the old ages : the same for the belgian population beyond the age of 76 (Brouhns & al. 2002)
- Mortality projection : with 4 components, the irregularity of Kt series is importante – use ARIMA time serie models to project (Li, Hardy and Tan)
- Dynamic life table allows more accurate calculations (reserving – pricing)

Thank you for your attention